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SPATIAL DISTRIBUTION OF TEMPERATURE AND TEMPERATURE INSTABILITY OF A THERMOELEMENT

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We show that, depending on the spatial distribution of temperature in semiconducting thermoelements, both "negative" (onset of temperature instability at negative temperature differences) and "positive" temperature instabilities may set in.

It is known [1, 2] that Peltier thermoelements are used not only for thermoelectric cooling of a current source but also as transformers of thermal or electric signals [1, 3].

It is easy to see that fluctuation of the thermal signal or variation of the temperature (for example, variation of the temperature of a working junction) leads to a change in the quantities that describe the thermoelectric properties of a semiconductor. For this reason, the object of our investigation was to calculate the spatial dependence of the temperature and the temperature instability of thermoelements. In contrast to [3], the Thomson effect was also taken into account along with the Peltier, Joule, and Seebeck effects. As a result, we obtained a thermal equation of the hydrodynamic type that describes the time-space dependence of the temperature in the form

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial \ln S}{\partial x} - \frac{\partial T}{\partial x} + \frac{J^2}{\chi\sigma} = \frac{1}{a}\frac{\partial T}{\partial t}, \tag{1}$$

where a is the coefficient of thermal diffusivity; $\chi_0 = \chi T^{n_\chi}$; χ is the thermal conductivity; $\sigma = \sigma_0 T^{n_\sigma}$; σ is the electrical conductivity of the semiconductor; n_χ and n_σ are numbers (powers) whose values depend on the mechanism of scattering of current carriers; $J = I/S$ is the current density. For the sake of simplicity, we assume that the coordinate dependence of the cross-sectional area is power-law, i.e., $S(x)S_0(1 + \beta x)^n$, where S_0 is the cross-sectional area of the "cold contact" (with the temperature T_{cold}), and β is a parameter that characterizes the spatial dependence of the cross-sectional area of the thermoelement; the x axis is selected in the direction of the increase in the temperature difference; T_{hot} is the temperature of the "hot" contact; $T(x = 0) = T_{\text{cold}}$, $T(x = l) = T_{\text{hot}}$, l is the distance between contacts with the temperatures T_{cold} and T_{hot}.

We note at once that an equation in the form (1) with account for $S(x)$ can be solved only with the aid of a computer, since at the present time we do not know experimental results that would allow us to compare them with numerical (computer) calculations. Therefore, we will limit ourselves to approximate analytical solutions. For this purpose, we will rewrite Eq. (1) taking into account the function $S(x)$. Then, the thermal equation of hydrodynamic type will take the following form for the stationary regime of a thermoelement:

$$y^{2n}T'' + ny^{2n-1}T' + \gamma T^m = 0. \tag{2}$$

Here $T' = \partial T/\partial y$, $T'' = \partial^2 T/\partial y^2$, $y = 1 + \beta x$, $\gamma = I^2/(S_0^2\sigma_0\chi_0)$. $m = -(n_\sigma + n_\chi)$.

The number m in the experimentally realizable temperature region of 200–300°C is equal to 3. Here, with an error smaller than 17% we may assume that $T^3 \approx \beta_T + \alpha_T T$, where $\beta_T = -3.09\cdot 10^{+7}\text{K}^3$, $\alpha_T = 1.88\cdot 10^{+5}\text{K}^2$. Then we rewrite Eq. (2) as

$$y^{2n}\theta'' + ny^{2n-1}\theta' + \tilde{\gamma}\theta \doteq 0, \tag{3}$$

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where $\tilde{\gamma} = \gamma\alpha_T$, $\theta = \beta_T + \alpha_T T$*. In this case we seek the solution of Eq. (3) in the form (see, for example, [4])

$$\theta_n = C_1 \sin\left(U_n \sqrt{\tilde{\gamma}}\right) + C_2 \cos\left(U_n \sqrt{\tilde{\gamma}}\right), \tag{4}$$

where $U_n = y^{1-n}/(n-1)$ $(n \neq 1)$. Equation (3) with $n = 1$ is called the Euler equation. It has the solution

$$\theta_1 = C_1 \sin\left(\ln|y| \sqrt{\tilde{\gamma}}\right) + C_2 \cos\left(\ln|y| \sqrt{\tilde{\gamma}}\right). \tag{3a}$$

We will determine the unknown coefficients C_1 and C_2 using the boundary conditions

$$\theta(x=0) \equiv \theta_{\text{cold}} = \alpha_T T_{\text{cold}} + \beta_T, \quad \theta(x=l) \equiv \theta_{\text{hot}} = \alpha_T T_{\text{hot}} + T\beta_T, \tag{5}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = \frac{\Pi j_0 + r j_0^2}{\chi(T_{\text{cold}})},$$

where Π is the Peltier coefficient on the working junction at $T = T_{\text{cold}}$; r is the electrical resistance of the region between the contacts, and we use the notation

$$\theta'_{\text{cold}} = \frac{\alpha_T}{\beta} \left.\frac{\partial T}{\partial x}\right|_{x=0}.$$

Then the solutions of Eqs. (3) and (3a) are written as

$$\theta_n = \frac{\theta_{\text{cold}} \sin\left[\frac{\sqrt{\tilde{\gamma}}}{n-1}\left(y^{1-n} - (1+\beta l)^{1-n}\right)\right] - \theta_{\text{hot}} \sin\left[\frac{\sqrt{\tilde{\gamma}}}{n-1}\left(y^{1-n} - 1\right)\right]}{\sin\left[\frac{\sqrt{\tilde{\gamma}}}{n-1}\left(1 - (1+\beta l)^{1-n}\right)\right]}, \tag{6}$$

$$\theta_1 = \frac{\theta_{\text{hot}} \sin\left(\sqrt{\tilde{\gamma}} \ln|y|\right) + \theta_{\text{cold}} \sin\left(\sqrt{\tilde{\gamma}} \ln\frac{|1+\beta l|}{|y|}\right)}{\sin\left(\sqrt{\tilde{\gamma}} \ln|1+\beta l|\right)}. \tag{6a}$$

From the last equation it is seen that, depending on the ratio $\theta'_{\text{cold}} l/\theta_{\text{cold}}$, the change in the temperature of the working junction may turn out to be negative, and therefore for $\delta T < 0$ the quantity δQ is also negative. A further decrease in the working-junction temperature leads to an increase in the quantity of absorbed heat, due to which the possibility of spatial-thermal instability appears.

Thus, in the case considered the spatial dependence of the temperature may exhibit, in certain regions of the values of $T(x)$, both "negative" (when the temperature difference takes negative values, with an increase in voltage up to a certain value $U = U_0$) and "positive" instability (when $U > U_0$) (see, for example, [3]).

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* Estimates show that for lead chalcogenide single crystals $\tilde{\gamma} < 0{,}25$.